15. Use separation of variables to solve the initial value problem.

$$\frac{dy}{dx} = e^{x-y} \quad \text{and} \quad y = 2 \text{ when } x = 0$$

$$\frac{dy}{dx} = e^x \cdot e^{-y}$$

$$\frac{dy}{e^{-y}} = e^x dx$$

$$\int e^y \, dy = \int e^x \, dx$$

$$e^y = e^x + C \qquad (0,2)$$

$$e^2 = e^0 + C$$

$$C = e^2 - 1$$

$$e^y = e^x + e^2 - 1$$

$$\boxed{y = \ln(e^x + e^2 - 1)}$$

1) $$\frac{dy}{dx} = \frac{x}{y} \qquad (1,2)$$

$$\int y \, dy = \int x \, dx$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + C$$

$$y^2 = x^2 + C$$

$$2^2 = 1^2 + C$$

$$C = 3$$

$$y^2 = x^2 + 3$$

$$\boxed{y = \sqrt{x^2+3}} \qquad (1,2)$$

(3) $\frac{dy}{dx} = \frac{y}{x}$ $(2,2)$

$\frac{1}{y}dy = \frac{1}{x}dx$

$\ln|y| = \ln|x| + C$

$\ln 2 = \ln 2 + C$

$C = 0$

$\ln y = \ln x$

$y = x$ $(0,\infty)$

(5) $\frac{dy}{dx} = (y+5)(x+2)$ $(0,1)$

$\int \frac{1}{y+5}dy = \int (x+2)dx$

$\ln|y+5| = \frac{1}{2}x^2 + 2x + C$ $(0,1)$

$\ln 6 = 0 + 0 + C$

$C = \ln 6$

$\ln|y+5| = \frac{1}{2}x^2 + 2x + \ln 6$

$\ln(y+5) = \frac{1}{2}x^2 + 2x + \ln 6$

$y+5 = e^{\frac{1}{2}x^2+2x+\ln 6}$

$y+5 = e^{\frac{1}{2}x^2+2x} \cdot e^{\ln 6}$

$y+5 = 6e^{\frac{1}{2}x^2+2x}$

$\boxed{y = 6e^{\frac{1}{2}x^2+2x} - 5}$

7. $\frac{dy}{dx} = (\cos x)e^{y+\sin x}$ $(0,-1)$

$\frac{dy}{dx} = (\cos x)\, e^{y} \cdot e^{\sin x}$

$\int e^{-y}\,dy = \int e^{\sin x}\cos x$

$-e^{-y} = e^{\sin x} + C$ $(0,-1)$

$-e^{1} = e^{\sin 0} + c$

$-e = 1 + c$

$c = -e - 1$

$-e^{-y} = e^{\sin x} - e - 1$

$e^{-y} = e + 1 - e^{\sin x}$

$-y = \ln(e + 1 - e^{\sin x})$

$y = -\ln(e + 1 - e^{\sin x})$

9. $\frac{dy}{dx} = -2xy^{2}$ $(1, \frac{1}{4})$

$\int y^{-2}\,dy = \int -2x\,dx$

$-y^{-1} = -x^{2} + C$

$-\frac{1}{y} = -x^{2} + C$

$\frac{1}{y} = x^{2} + C$ $(1, \frac{1}{4})$

$4 = 1^{2} + c$

$c = 3$

$\frac{1}{y} = x^{2} + 3$

$\boxed{y = \frac{1}{x^{2}+3}}$

Chapter 7 Review Part 2 Calculator Name_______________

16. Find the solution to the differential equation $\boxed{dy/dt = ky, \text{ k is a constant}}$ that satisfies the given conditions.

$y(0) = 60, \quad y(10) = 30$

$y = y_0 e^{kt}$

60 $\quad (10, 30)$ $\quad t \quad y$

$y = y_0 e^{kt}$

$30 = 60 e^{10k}$

$\frac{1}{2} = e^{10k}$

$\frac{\ln \frac{1}{2}}{10} = \frac{10k}{10}$

$k = -.0693$

$$\boxed{y = 60 e^{-.0693 t}}$$

17. Solve the problem.

In Exercises 15–18, complete the table for an investment if interest is compounded continuously.

	Initial Deposit ($)	Annual Rate (%)	Doubling Time (yr)	Amount in 30 yr ($)
15.	1000	8.6	8.06	13,197.14
16.	2000	4.62%	15	8,000
17.	600	5.25	13.2	2898.44
18.	1200	7.2%	9.63	10,405.37

$y = y_0 e^{kt}$

$A = Pe^{rt}$

(18) $y = y_0 e^{kt}$ $\quad A = Pe^{rt}$

1200 $\quad$ 10,405.37

$A = Pe^{rt}$

$\frac{10,405.37}{1200} = \frac{1200 e^{30r}}{1200}$

$\frac{10,405.37}{1200} = e^{30r}$

$\frac{\ln\left(\frac{10,405.37}{1200}\right)}{30} = \frac{30r}{30}$

$r = .0723 \quad 7.2\%$

$A = Pe^{rt} \qquad \frac{A}{P} = 2$

$2400 = 1200 e^{.072t}$

$2 = e^{.072t}$

$\frac{\ln 2}{.072} = \frac{.072t}{.072}$

$t = 9.6$ years

35 half life = 5700 years

$$y = y_0 \left(\tfrac{1}{2}\right)^{\frac{t}{5700}} \qquad \frac{y}{y_0} = .445$$

$$.445 = \left(\tfrac{1}{2}\right)^{\frac{t}{5700}}$$

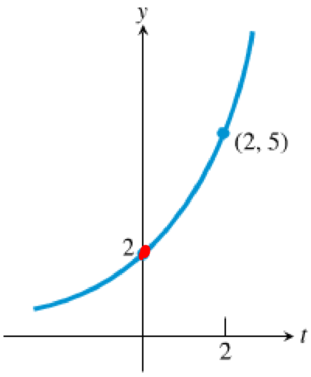
$$\frac{\ln .445}{\ln \frac{1}{2}} = \frac{\frac{t}{5700} \ln \frac{1}{2}}{\ln \frac{1}{2}}$$

$$1.168122 = \frac{t}{5700}$$

$$t = 6658$$

20. In Exercises 27 and 28, find the exponential function $y = y_0 e^{kt}$ whose graph passes through the two points.

27.



$$y = y_0 e^{kt}$$

$$y_0 = 2$$

$$(2, 5)$$

$$5 = 2e^{k(2)}$$

$$\frac{5}{2} = e^{2k}$$

$$\boxed{y = 2e^{.458t}}$$

$$\frac{\ln\left(\frac{5}{2}\right)}{2} = \frac{2k}{2} \qquad k = .458$$

18. ***Half-Life*** The radioactive decay of Sm-151 (an isotope of samarium) can be modeled by the differential equation $dy/dt = -0.0077y$, where t is measured in years. Find the half-life of Sm-151.

$$\frac{dy}{dt} = -.0077y$$

$$y = y_0e^{-.0077t}$$

$$\frac{dy}{dt} = ky$$

$$\downarrow$$

$$y = y_0e^{kt}$$

$$\frac{1}{2} = e^{-.0077t}$$

half life

$$\frac{y}{y_0} = \frac{1}{2}$$

$$\frac{\ln\frac{1}{2}}{-.0077} = \frac{-.0077t}{-.0077}$$

$$\boxed{t = 90 \text{ years}}$$