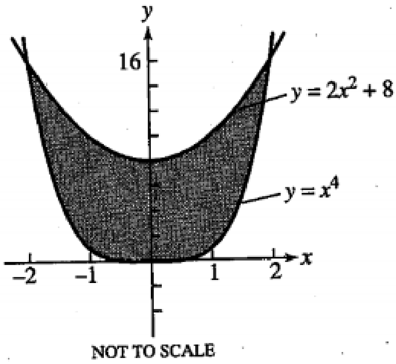
BC Chapter 8 Study Guide

1. What is the area of the shaded region shown?

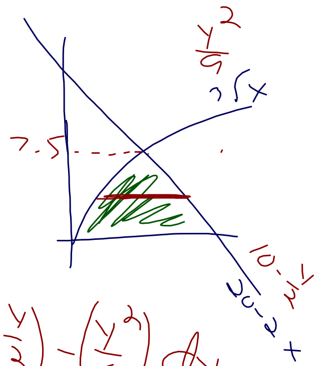


NOT TO SCALE

2. What is the area enclosed by $y = \sin x$ and the x-axis for $-2\pi \le x \le 2\pi$?

3. Find the area enclosed by $y = 3\sqrt{x}$, $y = 20 - 2x$, and the x-axis.

(A) 39.5736 (B) 45.3125 (C) 46.7382
(D) 49.7318 (E) 54.1402



$\int_0^{7.5} \left(10-\frac{y}{2}\right)-\left(\frac{y^2}{9}\right)dy$

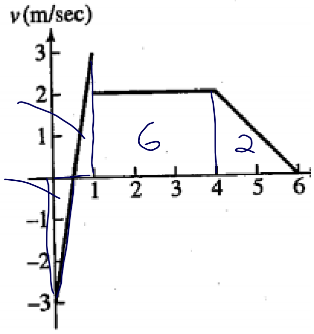
4. The function $v(t) = 9 - t^2$ is the velocity of a particle moving along the x-axis, where t is measured in seconds $(t \ge 0)$ and the velocity is measured in m/sec.

(a) Determine when the particle is moving to the right, to the left, and stopped.
(b) Find the particle's displacement for $0 \le t \le 6$.
(c) Find the total distance traveled by the particle for $0 \le t \le 6$.

5. The graph shows the velocity of a particle moving on the x-axis. The particle starts at $x = -5$ when $t = 0$.

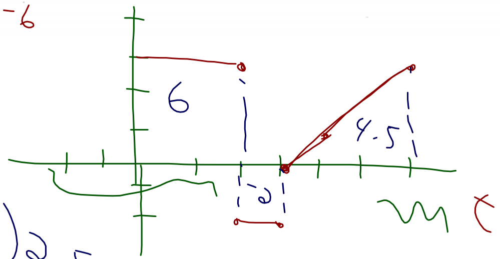
(a) Find where the particle is at the end of the trip $(t = 6)$. $-5+8=3$
(b) Find the total distance traveled by the particle.

$\frac{3}{4}$ $+\frac{3}{4}$



$\frac{3}{4}+\frac{3}{4}+6+2=$

$t=0$
$x=-6$
d



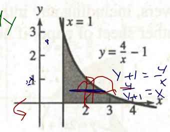
a) 2.5
b) 12.5

6. The rate of expenditures on public elementary and secondary schools, in billions of dollars per year, in a region of the United States can be modeled by the function $S = 6.81e^{0.082t}$, where t is the number of years after January 1, 1980. What are the total expenditures from January 1, 1980 to January 1, 2005 for this model?

Shell $2\pi \int_0^3 y(x-1)\,dy$ πr^2 Disk

7. What is the volume generated by revolving the shaded region around the x-axis? Use disks.

$2\pi \int_0^3 y\left(\frac{4}{y+1} - 1\right) dy$



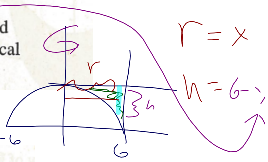
$\pi \int_1^4 y^2\,dx$

$\pi \int_1^4 \left(\frac{4}{x} - 1\right)^2 dx$

8. A region bounded by $y = \sqrt{36 - x^2}$, $y = 6$, and $x = 6$ is revolved around the y-axis. Use cylindrical shells to find the volume of the solid generated.

$r = x$

$h = 6 - \sqrt{\ }$



$2\pi \int r\,h\,dx = 2\pi \int_0^6 x\left(6 - \sqrt{36 - x^2}\right) dx$

9. Find the area of the surface generated by revolving the curve $y = \frac{x^3}{9}$, $0 < x < 2$, about the x-axis. $y' = \frac{1}{3}x^2$

$\int 2\pi r \sqrt{1 + (f'(x))^2}\,dx$

$2\pi \int_0^2 (y) \sqrt{1 + \left(\frac{x^2}{3}\right)^2}\,dx$

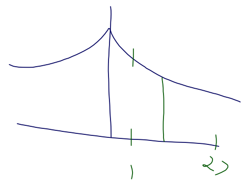
$2\pi \int_0^2 \frac{x^3}{9} \sqrt{1 + \frac{x^4}{9}}\,dx$

10. A curve is given by $y = (16 - x^{2/3})^{3/2}$ for $1 \le x \le 27$. Find the exact length of the curve analytically by antidifferentiation.

$f'(x) = \frac{3}{2}(16 - x^{2/3})^{\frac{1}{2}} \cdot \frac{2}{3}x^{-\frac{1}{3}}$

$\int_a^b \sqrt{1 + (f'(x))^2}\,dx$

$\int_1^{27} \sqrt{1 + \left((16 - x^{2/3})^{\frac{1}{2}} \cdot x^{-\frac{1}{3}}\right)^2}\,dx$



$\int_1^{27} \sqrt{1 + (16 - x^{2/3})\,x^{-2/3}}\,dx$

$\int_1^{27} \sqrt{1 + 16x^{-2/3} - x^0}$

$\int_1^{27} \sqrt{16x^{-2/3}} =$

$4\int_1^{27} \left(x^{-\frac{2}{3}}\right)^{\frac{1}{2}} dx$

$4\int_1^{27} x^{-\frac{1}{3}}\,dx$

$4\left[\frac{3}{2}x^{2/3}\right]_1^{27}$

$4\left[\frac{3}{2}(9) - \frac{3}{2}\left(\frac{3}{2}\right)\right]$

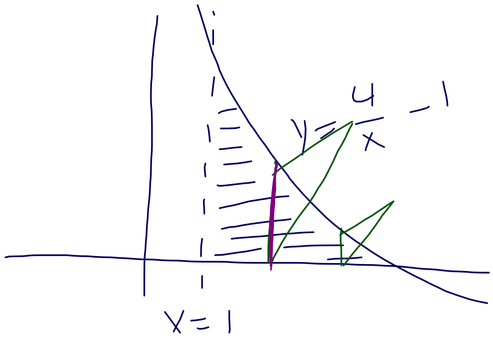
$4\left[\frac{27}{2} - \frac{9}{4}\right]$

$4\left[\frac{45}{4}\right]$

?

45

7.5



Volume using equilateral triangles as our cross sections $\perp$ to x-axis.

$$A(x) = \frac{\sqrt{3}}{4}(\text{side})^2$$

$$\int_1^4 \frac{\sqrt{3}}{4}(\text{side})^2\,dx$$

$$\int_1^4 \frac{\sqrt{3}}{4}\left(\frac{4}{x} - 1\right)^2 dx$$

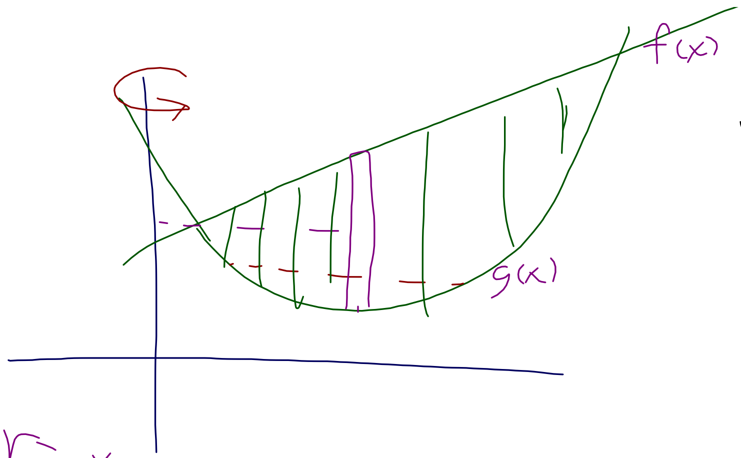
of semicircles

$$\frac{1}{2}\pi\int_1^4 \left(\frac{1}{2}\left(\frac{4}{x} - 1\right)\right)^2 dx$$

of squares

$$\int_1^4 \left(\frac{4}{x} - 1\right)^2 dx$$

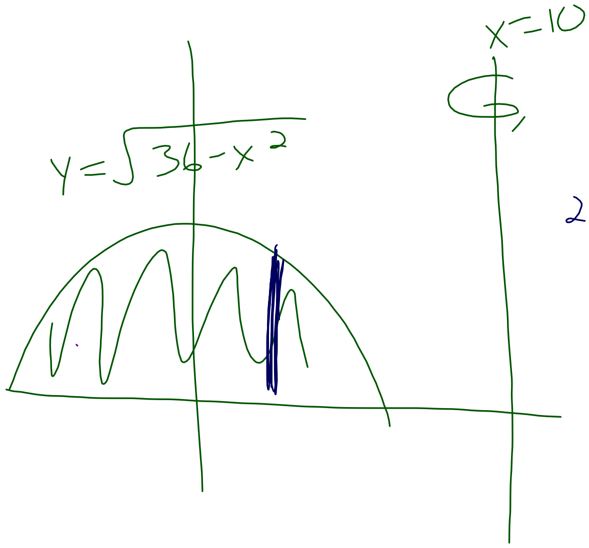
Shell



$r = x$

$h = f(x) - g(x)$

$$2\pi \int r \cdot h \, dx$$



$$2\pi \int_{-6}^{6} \underbrace{(10-x)}_{r} \underbrace{\left(\sqrt{36-x^2}\right)}_{h} \, dx$$

Shell

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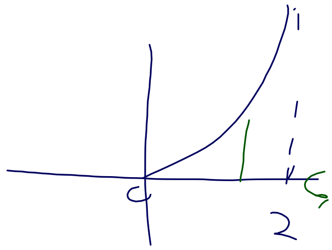
~~A curve is given by $y = (16 - x^{2/3})^{3/2}$ for $1 \le x \le 27$. Find the exact length of the curve analytically by antidifferentiation.~~



$$y' = \frac{1}{3}x^2$$

$$r = y$$

$$2\pi \int y \sqrt{1 + \left(\tfrac{1}{3}x^2\right)^2}\, dx$$

$$2\pi \int_0^2 \frac{x^3}{9} \sqrt{1 + \frac{x^4}{9}}\, dx$$

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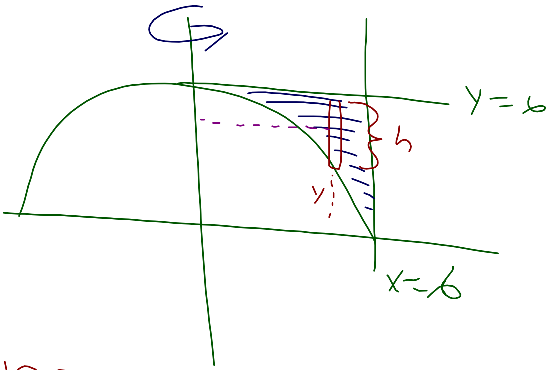

$r =$

$h =$

$$2\pi \int_a^b r \quad h \quad d(\quad)$$

$$2\pi \int_0^6 x\,(6-y)\,dx$$

$$2\pi \int_0^6 x\left(6-\sqrt{36-x^2}\right)dx$$