Study Guide Chapter 9

1) D	10) 0^0 1	19) B
2) C	11) A	
3) B	12) B	
4) A	13) x^3, $(\ln 3)^x$, e^x, 3^x	
5) C	14) $\lim_{c\to 8^-}\int_0^c \frac{dx}{64-x^2}+\lim_{c\to 8^+}\int_c^{16}\frac{dx}{64-x^2}$	
6) D	15) Diverges	
7) $a_1 = 1/16$ $r = -4$ $a_n = (-1)^{n-1}4^{n-3}$	16) A	
8) B	17) D	
9) C	18) D	

1) $(-1)^0 \cdot 2 = +2$

$(-1)^1 \cdot 8 = -8$

2) $a_1 = 1$

$a_2 = 6$

$a_n = a_{n-1} - a_{n-2}$

$a_3 = a_2 - a_1$

$= 6 - 1 = 5$

$a_4 = a_3 - a_2$

$= 5 - 6$

$= -1$

#4) 52, 20.8, 8.32, 3.328, 1.33, ...

$20.8 - 52 = 8.32 - 20.8$

$31.2 \neq 12.48$

not arithm.

$\frac{20.8}{52} = \frac{8.32}{20.8}$

$.4 = .4$

$\therefore$ Geometric with $r = .4$

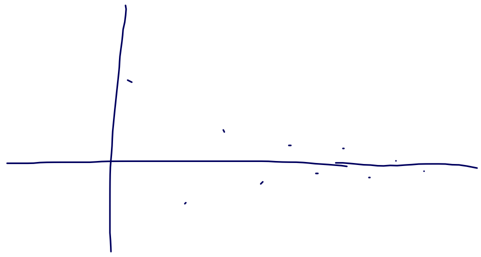
explicit form $g_n = g_1 \cdot (.4)^{n-1}$

recursive $g_1 = 52$

$g_n = g_{n-1}(.4)$

#5) $a_n = (-1)^n \left(\frac{n+9}{n^2+8}\right)$

$$\lim_{n\to\infty}\left(\frac{n+9}{n^2+8}\right) = 0$$



0

#6) $a_n = \frac{4n+8}{7+1\sqrt{n}}$

$$\lim_{n\to\infty}\frac{4n+8}{7+1\sqrt{n}} = \infty$$

$\therefore$ Divergent.

$$\frac{4}{\frac{1}{2}n^{-\frac{1}{2}}} = \cancel{4}\ 8\sqrt{n}$$

#7) $g_2 = -\frac{1}{4}$ $g_5 = 16$

$g_n = g_1 \cdot r^{n-1}$

$-\frac{1}{4} = g_1 \cdot r^{2-1}$ $16 = g_1 \cdot r^{5-1}$

$$\div \frac{16 = \cancel{g_1} \cdot r^4}{-\frac{1}{4} = \cancel{g_1} \cdot r^1}$$

$-64 = r^3$

$r = -4$

$-\frac{1}{4} = g_1 \cdot (-4)$

$\frac{1}{16} = g_1$

$\boxed{g_n = \frac{1}{16} \cdot (-4)^{n-1}}$

$g_n = 4^{-2} \cdot (-4)^{n-1}$

$g_1 = \frac{1}{16}$

$g_n = g_{n-1} \cdot (-4)$

$g_n = 4^{-2} (-1)^{n-1} \cdot 4^{n-1}$

$g_n = 4^{n-3}(-1)^{n-1}$

#9) $\lim_{x \to 3} \frac{\sqrt{6+x} - 3}{x-3} = \frac{0}{0}$

$\lim_{x \to 3} \frac{\frac{1}{2}(6+x)^{-\frac{1}{2}}}{1}$

$\lim_{x \to 3} \frac{1}{2\sqrt{6+x}} = \frac{1}{6}$

10) $\lim_{x \to 0^+} (\tan x)^x$

$f(x) = \tan x^x$

$\ln f(x) = x \ln \tan x$

$e^{\ln f(x)} = e^0$

$f(x) = e^0 = 1$

$\lim_{x \to 0^+} \frac{\ln(\tan x)}{\frac{1}{x}}$ $\quad \frac{-\infty}{\infty}$

$\lim_{x \to 0^+} \frac{\frac{1}{\tan x} \cdot \sec^2 x}{-\frac{1}{x^2}}$

$\frac{\cancel{\cos x}}{\sin x} \cdot \frac{1}{\cos^{\cancel{2}} x} \cdot \frac{-x^2}{1}$

$\lim_{x \to 0^+} \frac{-x^2}{\sin x \cos x}$ $\quad \frac{0}{0}$

$\lim_{x \to 0^+} \frac{-2x}{\cos x \cos x + \sin x(-\sin x)}$

$\lim_{x \to 0^+} \frac{-2x}{\cos^2 x - \sin^2 x} = \cos 2x$

0

11) $\lim_{x \to \infty} \frac{\ln x^5}{x} = \lim_{x \to \infty} \frac{\frac{1}{x^5} \cdot 5x^4}{1}$

$= \lim_{x \to \infty} \frac{5}{x}$

$= 0$

13) $x^3,\ e^x,\ 3^x,\ (\ln 3)^x$

$x^3,\ (\ln 3)^x,\ e^x,\ 3^x$

5^{-x} $\quad 5^{-x}\ln 5$

$$\lim_{x\to\infty}\frac{(\ln 3)^x}{x^3}=\lim_{x\to\infty}\frac{(\ln 3)^x\ln(\ln 3)}{3x^2}=\lim_{x\to\infty}\frac{(\ln 3)^x[\ln(\ln 3)]^2}{6x}$$

$$\lim_{x\to\infty}\frac{3^x}{e^x}=\lim_{x\to\infty}\left(\frac{3}{e}\right)^x=\infty$$

$7^x+5^x \qquad 7^x$

$$\lim_{x\to\infty}\frac{7^x+5^x}{7^x}$$

$$\frac{7^x}{7^x}+\frac{5^x}{7^x}$$

$$1+\left(\frac{5}{7}\right)^x$$

$$\lim_{x\to\infty} \frac{x^3}{e^x} = \lim_{x\to\infty} \frac{3x^2}{e^x} = \lim_{x\to\infty} \frac{6x}{e^x} = \lim_{x\to\infty} \frac{6}{e^x} = 0$$

$$\lim_{x\to\infty} \frac{e^x}{3^x} = \lim_{x\to\infty} \left(\frac{e}{3}\right)^x = \lim_{x\to\infty} \left(\begin{matrix}\text{exp.}\\ \text{decay}\end{matrix}\right) = 0$$

$$\frac{e^x}{3^x \ln 3} \qquad \frac{e^x}{3^x (\ln 3)^2} \qquad \left(\frac{e}{3}\right)^x$$

$$3^x \qquad 5^x$$

$$\lim_{x\to\infty} \frac{3^x}{5^x} = \lim_{x\to 3} \left(\frac{3}{5}\right)^x = 0$$

#15) $\int_1^{\infty} \frac{6+\cos x}{\sqrt{x}}\,dx$

$\lim_{b\to\infty} \int_1^{b} \frac{\overset{5\leftrightarrow 7}{6+\overset{-1\leftrightarrow 1}{\cos x}}}{\sqrt{x}}\,dx$

$\frac{5}{\sqrt{x}} < \frac{6+\cos x}{\sqrt{x}} < \frac{7}{\sqrt{x}}$ $\Big\{$ $\frac{6+\cos x}{\sqrt{x}} > \frac{1}{\sqrt{x}}$

$\lim_{b\to\infty} \int_1^{b} \frac{1}{\sqrt{x}}\,dx$

$\lim_{b\to\infty} \left[2x^{\frac{1}{2}}\right]_1^b$

$\lim_{b\to\infty} \left[\cancel{2\sqrt{b}} - 2\right]$

Diverges

#16) $\int_0^{25} \frac{1}{\sqrt{|x-9|}}\,dx$ $\quad \begin{cases} x-9 & x\geq 9 \\ 9-x & x<9 \end{cases}$

$\lim_{b\to 9^-} \int_0^{b} \frac{1}{\sqrt{9-x}}\,dx + \lim_{a\to 9^+} \int_a^{25} \frac{1}{\sqrt{x-9}}\,dx$

$\lim_{b\to 9^-} \left[-2\sqrt{9-x}\right]_0^b + \lim_{a\to 9^+} \left[2\sqrt{x-9}\right]_a^{25}$

$\lim_{b\to 9^-} \left[-2\sqrt{9-b} + 6\right] + \lim_{a\to 9^+} \left[8 - 2\sqrt{a-9}\right]$

$6 + 8 = 14$

17) $\int_0^{\ln 4} \frac{1 e^{\frac{1}{y^2}}}{y^3} dy$

$u = \frac{1}{y^2}$ y^{-2}

$du = -\frac{2}{y^3} dy$

$-\frac{1}{2} du = \frac{1}{y^3} dy$

$-\frac{1}{2}\int e^u \, du$

$-\frac{1}{2} e^u$

$-\frac{1}{2} e^{\frac{1}{y^2}}$

$\lim_{b \to 0^+} -\frac{1}{2}\left[e^{\frac{1}{y^2}}\right]_b^{\ln 4}$

$\lim_{b \to 0^+}\left[-\frac{1}{2} e^{\frac{1}{(\ln 4)^2}} - -\frac{1}{2} e^{\frac{1}{b^2}}\right]$

Diverges

(18) $\int_0^\infty \frac{25x}{1+x^2} dx$

$u = 1 + x^2$ $du = 2x\, dx$

$\frac{1}{2} du = x\, dx$

$\frac{25}{2}\int \frac{1}{u} du$

$\frac{25}{2}\left[\ln u\right]$

$\lim_{b \to \infty} \frac{25}{2}\left[\ln(1+x^2)\right]_0^b$

$\lim_{b \to \infty} \frac{25}{2}\left[\ln(1+b^2) - \ln(1-0^2)\right]$

$\lim_{b \to \infty} \frac{25}{2}\left[\ln(1+b^2) - 0\right]$

∞

Div.

19) $\int_{-\infty}^{0} x^2 e^{3x}\,dx$

$\lim_{b\to-\infty} \int_{b}^{0} x^2 e^{3x}\,dx$

	u	dv
+	x^2	e^{3x}
−	$2x$	$\frac{1}{3}e^{3x}$
+	2	$\frac{1}{9}e^{3x}$
	0	$\frac{1}{27}e^{3x}$

$\lim_{b\to-\infty} \left[\frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x}\right]_b^0$

$\lim_{b\to-\infty} \left[\frac{2}{27} - \left(\underbrace{\frac{1}{3}b^2e^{3b}}_{0} - \underbrace{\frac{2}{9}be^{3b}}_{0} + \underbrace{\frac{2}{27}e^{3b}}_{0}\right)\right]$