Study Guide...

# 7.1

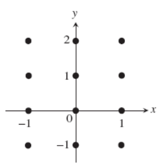
In Exercises 1–10, find the general solution to the exact differential equation.

1. $\dfrac{dy}{dx} = 5x^4 - \sec^2 x$

2. $\dfrac{dy}{dx} = \sec x \tan x - e^x$

14. $\dfrac{dA}{dx} = 10x^9 + 5x^4 - 2x + 4$ and $A = 6$ when $x = 1$

In Exercises 29–34, construct a slope field for the differential equation. In each case, copy the graph at the right and draw tiny segments through the twelve lattice points shown in the graph. Use slope analysis, not your graphing calculator.




33. $\dfrac{dy}{dx} = x + 2y$

# 7.2

3. $\int \left(t^2 - \frac{1}{t^2}\right)$

5. $\int (3x^4 - 2x^{-3} + \sec^2 x)\, dx$

41. $\int \frac{x\, dx}{x^2 + 1}$

59. $\int_{t=0}^{t=1} \sqrt{t^5 + 2t}\,(5t^4 + 2)\, dt$

$u = t^5 + 2t \qquad du = 5t^4 + 2\, dt$

$$\int_{u=0}^{u=3} \sqrt{u}\, du$$

$$\left[\frac{2}{3}u^{\frac{3}{2}}\right]_0^3 = \frac{2}{3}(3)^{3/2} - 0$$

**66.** $\displaystyle\int_0^2 \frac{e^x\,dx}{3+e^x}$

7.3

**5.** $\displaystyle\int x^2 \cos x\,dx$

**22.** $\displaystyle\int (x^2-5x)e^x\,dx$

9. $\frac{dy}{dx} = -2xy^2$ and $y = 0.25$ when $x = 1$

# 7.4

24. ***Bacteria Growth*** A colony of bacteria is grown under ideal conditions in a laboratory so that the population increases exponentially with time. At the end of 3 h there are 10,000 bacteria. At the end of 5 h there are 40,000 bacteria. How many bacteria were present initially?

$(3, 10000)$ $(3, 1)$

$(5, 40000)$ $(5, 4)$

$\int \sin x \sin x \, dx$

$\cancel{\int \sin x\, dx \int \sin x \, dx}$

$B = B_0 (b)^{kt}$

$40000 = B_0 (b)^{5k}$

$10000 = B_0 (b)^{3k}$

$4 = b^{2k}$ $b = e$

$4 = e^{2k}$

$\frac{\ln 4}{2} = k$

$B = \boxed{B_0} e^{\frac{\ln 4}{2} t}$

$10000 = B_0 e^{\frac{3 \ln 4}{2}}$

$B_0 = \frac{10000}{e^{\frac{3 \ln 4}{2}}}$

$$\int \sin^2 x \cos^3 x \, dx$$

$$\int \sin^2 x \cos^2 x \cos x \, dx$$

$$\int \sin^2 x \left(1-\sin^2 x\right) \cos x \, dx$$

$u = \sin x \qquad du = \cos x \, dx$

$$\int u^2\left(1-u^2\right) du$$

$$\int u^2 - u^4 \, du$$

$$\frac{1}{3}u^3 - \frac{1}{5}u^5 + C$$

$$\int \ln x \, dx$$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x} dx \qquad v = x$

$x \ln x - \int 1 \, dx$

$x \ln x - x + C$

| $(x,y)$ | $\frac{dy}{dx} = x-y$ | $\Delta x$ | $\Delta y = \frac{dy}{dx}\Delta x$ | $(x+\Delta x, y+\Delta y)$ |
|---|---|---|---|---|
| $(2,3)$ | $-1$ | $.5$ | $-1 \cdot .5 = -.5$ | $(2.5, 2.5$ |
| $(2.5, 2.5)$ | | $.5$ | | |
| | | $.5$ | | |
| $(3.5, \quad )$ | | | | |

$$\int \sin^3 x \cos x \, dx$$

$u = \sin x \qquad du = \cos x \, dx$

$$\int u^3 \, du$$

$\frac{1}{4}u^4 \qquad \frac{1}{4}\sin^4 x + C$

$\sin^2 x + \cos^2 x = 1$

$\frac{1}{4}\left(\sin^2 x\right)^2$

$\frac{1}{4}\left(1-\cos^2 x\right)^2$

$\frac{1}{4}\left(1-2\cos^2 x + \cos^4 x\right)$

$\frac{1}{4} - \frac{1}{2}\cos^2 x + \frac{1}{4}\cos^4 x$

$$T - T_s = (T_0 - T_s)e^{-kt}$$

$$T - 70 = T_0 - 7_0\ e^{-kt}$$

$$220.7 \quad 290.7 - 70 = (T_0 - 70)e^{-7k}$$

$$\div 174 \quad 244 - 70 = (T_0 - 70)e^{-14k}$$

$$1.268390805 = e^{7k}$$

$$k = \frac{\ln 1.268}{7}$$

$$174 = (T_0 - 70)e^{-14(k)}$$

(2)

$$I = A_0\left(\tfrac{1}{2}\right)^{\frac{t}{1750}}$$

$$.2A_0 = A_0\ \tfrac{1}{2}^{t/1750}$$

$$.2 = \tfrac{1}{2}^{t/1750}$$

$$\ln .2 = \frac{t}{1750}\ln\frac{1}{2}$$

$$\frac{1750\ \ln(.2)}{\ln(\frac{1}{2})} = t$$

**26.** ***Polonium-210*** The number of radioactive atoms remaining after $t$ days in a sample of polonium-210 that starts with $y_0$ radioactive atoms is $y = y_0e^{-0.005t}$

**(a)** Find the element's half-life.

**(b)** Your sample will not be useful to you after 95% of the radioactive nuclei present on the day the sample arrives have disintegrated. For about how many days after the sample arrives will you be able to use the polonium?

a)

$$\tfrac{1}{2}y_0 = y_0e^{-.005t}$$

$$\tfrac{1}{2} = e^{-.005t}$$

$$\ln\tfrac{1}{2} = -.005t$$

$$t = \frac{\ln\frac{1}{2}}{-.005}$$

b)

$$.05 = e^{-.005t}$$

$\frac{1}{2}$ 2 3

$$P = y_0(\quad)^{kt}$$

$$P = y_0\left(\tfrac{1}{2}\right)^{\frac{t}{L}}$$

$L \rightarrow$ length of half life

(6) $\frac{dP}{dt} = .0006P(1900 - P)$

$\frac{1900}{.0006}$ (0, 13)

$P = \frac{1900}{1 + A_0 e^{-1.14t}}$

$13 = \frac{1900}{1 + A_0}$

# 7.5

**20.** $\int \frac{5x+2}{2x^2+x-1}dx$

In Exercises 23–26, the logistic equation describes the growth of a population $P$, where $t$ is measured in years. In each case, find **(a)** the carrying capacity of the population, **(b)** the size of the population when it is growing the fastest, and **(c)** the rate at which the population is growing when it is growing the fastest.

**24.** $\frac{dP}{dt} = 0.0008P(700 - P)$

**34.** ***Gorilla Population*** A certain wild animal preserve can support no more than 250 lowland gorillas. Twenty-eight gorillas were known to be in the preserve in 1970. Assume that the rate of growth of the population is

$$\int \frac{dP}{dt} = \overset{K}{0.0004}P(\overset{m}{250} - P),$$

$$P = \frac{M}{1 + A_0 e^{-(MK)t}}$$

where time $t$ is in years.

**(a)** Find a formula for the gorilla population in terms of $t$.

**(b)** How long will it take for the gorilla population to reach the carrying capacity of the preserve?

249.9

249.99

249

$(0, 28)$

$$P = \frac{250}{1 + A_0 e^{-.1t}}$$

$$28 = \frac{250}{1 + A_0 e^{-.1(0)}}$$

$A_0 = 7.928$