$$\int_1^{\infty} \frac{5x+6}{x^2+2x}\,dx$$

$$\frac{A}{x(x+2)}$$

$$\lim_{b\to\infty} \boxed{\int_1^{b} \frac{5x+6}{x^2+2x}\,dx}$$

$$5\int_1^b \frac{x}{x^2+2x}\,dx + \int_1^b \frac{6}{\underset{x(x+2)}{x^2+2x}}\,dx$$

$$5\int_1^b \frac{1}{x+2}\,dx$$

$u = x^2+2x$

$du = 2x+2\,dx$

$5[\ln|b+2| - \ln 3]$

MM

$$\int_{-1}^{\infty} \frac{1}{\underset{(x+3)(x+2)}{x^2+5x+6}}\,dx$$

$$\frac{1}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$$

$1 = A(x+3) + B(x+2)$

$1 = B(-1)$

$B = -1 \quad A = 1$

$$\lim_{b\to\infty} \int_{-1}^{b} \frac{1}{x+2} - \frac{1}{x+3}\,dx$$

$\ln \frac{x+2}{x+3}$

$$\lim_{b\to\infty} \left[\ln|x+2| - \ln|x+3|\right]_{-1}^{b}$$

$$\lim_{b\to\infty} \frac{b+2}{b+3}$$

$$\lim_{b\to\infty} \left[\ln\left(\frac{b+2}{b+3}\right) - \ln\left(\frac{1}{2}\right)\right]$$

$0 - \ln(2)^{-1}$

$\ln 2$

$$\int_2^\infty \frac{3}{x^2-x}$$
$(x)(x-1)$

$$\frac{3}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$$

$$3 = A(x-1) + B(x)$$

$B=3 \quad A=-3$

$$\lim_{b\to\infty} \int_2^b \frac{-3}{x} + \frac{3}{x-1}\, dx$$

$$-3\ln x + 3\ln|x-1|$$

$$\int_1^\infty x \ln x \, dx \qquad uv - \int v\,du \qquad x\ln x - x$$

$u = \ln x \quad dv = x\,dx$

$du = \frac{1}{x}dx \quad v = \frac{1}{2}x^2$

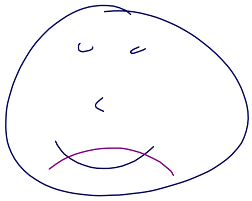
$$\lim_{a\to\infty} \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x\, dx$$

$$\lim_{a\to\infty} \left[\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2\right]_1^a$$

$$\left(\frac{1}{2}a^2 \ln a - \frac{1}{4}a^2\right) - \left[\frac{1}{2}(1)^2 \ln(1) - \frac{1}{4}(1)^2\right]$$

DIV

(32) $\int_1^\infty \frac{1}{x^3+1}\,dx$



$\lim_{b\to\infty}\int_1^b \frac{1}{x^3+1}\,d$

$0 \le \frac{1}{x^3+1} \le \frac{1}{2}$

$0 \le \frac{1}{x^3+1} \le \frac{1}{x^3}$

$\lim_{b\to\infty}\int_1^b \frac{1}{x^3}\,dx = \lim_{b\to\infty} -\frac{1}{2x^2}\Big]_1^b$

$\lim_{b\to\infty} -\frac{1}{2b^2} - \frac{-1}{2}$

$\frac{1}{2}$

30) $\int_{-1}^4 \frac{1}{\sqrt{|x|}}\,dx$

$|x| = \begin{cases} x & x \ge 0 \\ -x & x < 0 \end{cases}$

$\lim_{b\to 0^-}\int_{-1}^{b} \frac{1}{\sqrt{-x}} + \lim_{a\to 0^+}\int_a^4 \frac{1}{\sqrt{x}}\,dx$

28 $\displaystyle\int_0^4 \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$

$u = -\sqrt{x}$

$du = -\frac{1}{2}x^{-\frac{1}{2}}dx$

$= \frac{-1}{2\sqrt{x}}dx$

$u = -\sqrt{x}$

$du = -\frac{1}{2}x^{-\frac{1}{2}}dx$

$du = \frac{-1}{2\sqrt{x}}dx$

$-2du = \frac{1}{\sqrt{x}}dx$

$\displaystyle\lim_{b\to 0^+} -2\int_b^4 e^u\,du$

$\displaystyle\lim_{b\to 0^+} -2\left[e^u\right]_{x=b}^{x=4}$

$\displaystyle\lim_{b\to 0^+} -2\left[e^{-\sqrt{x}}\right]_b^4$

$\displaystyle\lim_{b\to 0^+} -2e^{-2} + 2e^{-\sqrt{b}}$

$-2e^{2} + 2$

$\displaystyle\int_{-\infty}^{\infty} \frac{1}{e^x + e^{-x}}\,dx$

$\displaystyle\frac{1}{e^x + \frac{1}{e^x}}\,dx$

$\displaystyle\frac{e^x}{(e^x)^2+1}$

$\displaystyle\frac{e^x}{e^{2x}+1}\,dx$

$(e^x)^2$

$\displaystyle\int_{-\infty}^{\infty} \frac{e^x}{e^{2x}+1}\,dx$

$u = e^x \qquad du = e^x dx$

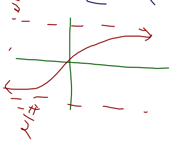
$\displaystyle\int_{x=-\infty}^{x=\infty} \frac{1}{u^2+1}\,du$

$\displaystyle\lim_{b\to -\infty}\int_b^0 \frac{1}{u^2+1}\,du + \lim_{a\to\infty}\int_0^a \frac{1}{u^2+1}\,du$

$\tan^{-1}(u)$

$\displaystyle\lim_{b\to -\infty}\left[\tan^{-1}(e^x)\right]_b^0 + \lim_{a\to\infty}\left[\tan^{-1}(e^x)\right]_0^a$

$\displaystyle\lim_{b\to -\infty}\left[\tan^{-1}(e^0) - \tan^{-1}e^b\right] + \lim_{a\to\infty}\left[\tan^{-1}e^a - \tan^{-1}e^0\right]$



$\frac{\pi}{4} - 0 + \frac{\pi}{2} - \frac{\pi}{4}$

22 $\int_{-\infty}^{\infty} 2x e^{-x^2} dx$

$u = -x^2$

$du = -2x\,dx$

$-du = 2x\,dx$

$\lim_{b \to -\infty} \int_b^0 \cancel{2x} e^{-x^2} \cancel{dx} + \lim_{a \to \infty} \int_0^a 2x e^{-x^2} dx$

$-\int e^u du$

$-e^{-x^2}$

$\lim_{b \to -\infty} \left[-e^0 + e^{-b^2} \right]_b^0$ $\quad \frac{1}{e^{b^2}}$

$\left[-1 + 0 \right]$

$\int_1^{\infty} \frac{1}{x^3+1} dx$

$x^2\left(x + \frac{1}{x^2}\right)$

$\frac{1}{x^2} + \frac{1}{x + \frac{1}{x^2}}$ $\quad \frac{x^2}{x^3+1}$

$0 \le \int_1^{\infty} \frac{1}{x^3+1} dx \le \int_1^{\infty} \frac{1}{x^3} dx$

$\lim_{b \to \infty} \left[-\frac{1}{2x^2} \right]_1^b$

$\lim_{b \to \infty} \left[-\frac{1}{2b^2} - -\frac{1}{2} \right] = \frac{1}{2}$

$$\int_{-1}^{4} \frac{1}{\sqrt{|x|}}\,dx$$

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$\int_{-1}^{0} \frac{1}{\sqrt{-x}} \qquad \int_{0}^{4} \frac{1}{\sqrt{x}}\,dx$$

$$\lim_{b \to 0^-} \int_{-1}^{b} \frac{1}{\sqrt{-x}}\,dx + \lim_{a \to 0^+} \int_{a}^{4} \frac{1}{\sqrt{x}}\,dx$$

26, 31, 33, 37, 39, 40, 42, 48–52