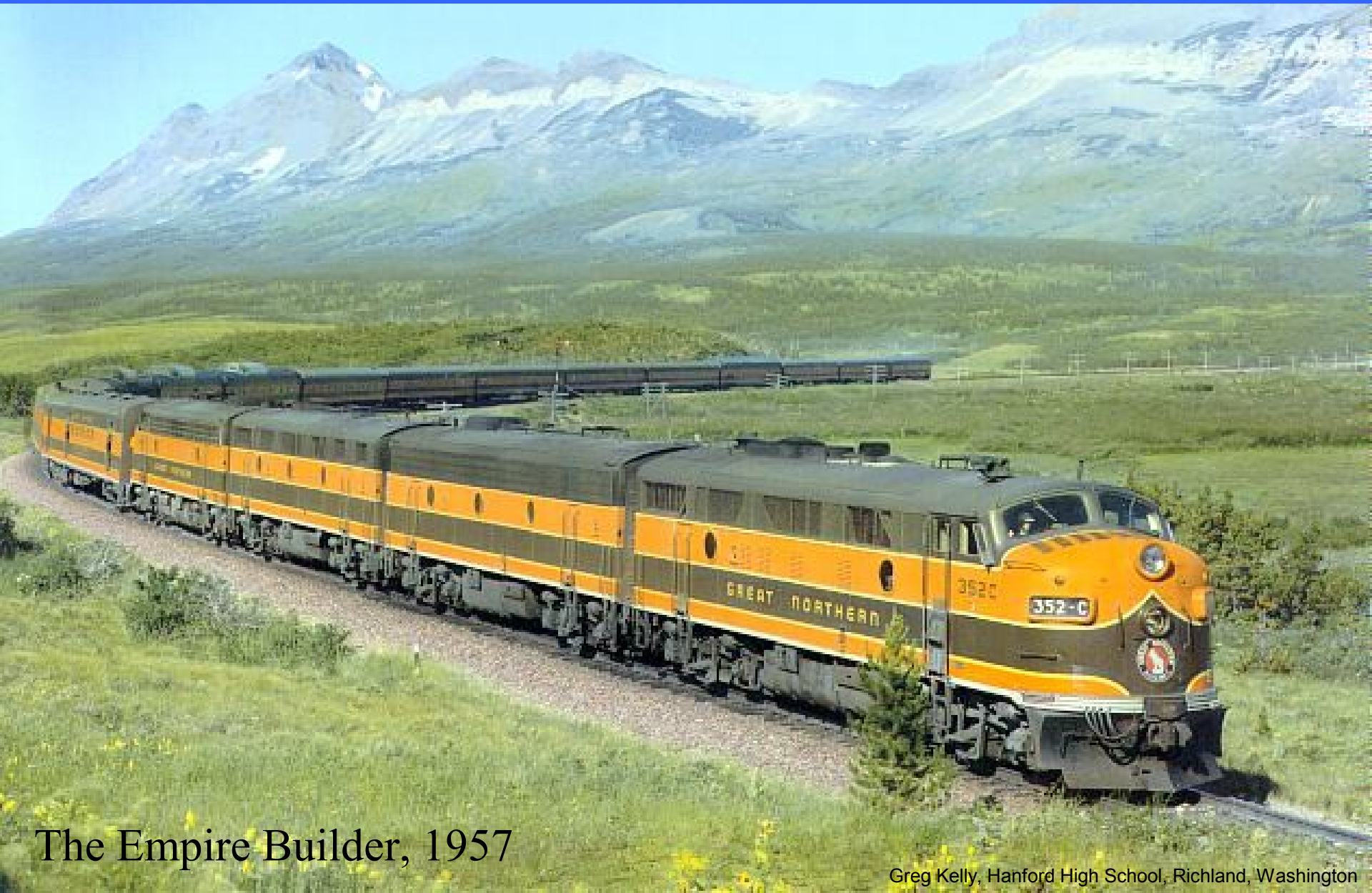
8.4 Partial Fractions



The Empire Builder, 1957

1

$$\int \frac{5x-3}{x^2-2x-3}\,dx$$

This would be a lot easier if we could re-write it as two separate terms.

$$\frac{5x-3}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1}$$

Multiply by the common denominator.

$$5x-3 = A(x+1)+B(x-3)$$

$$5x-3 = Ax+A+Bx-B\cdot 3$$

Set like-terms equal to each other.

$$5x = Ax+Bx \qquad -3 = A-B\cdot 3$$

$$5 = A+B \qquad -3 = A-3B$$

Solve two equations with two unknowns.

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1

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$$\frac{5x-3}{(x-3)(x+1)} = \frac{A}{x-3}+\frac{B}{x+1}$$

$$5x-3 = A(x+1)+B(x-3)$$

$$5x-3 = Ax+A+Bx-B\cdot 3$$

$$5x = Ax+Bx \qquad -3 = A-B\cdot 3$$

$$5 = A+B \qquad -3 = A-3B$$

$$5 = A+B \qquad -3 = A-3B$$

$$3 = -A+3B$$

$$8 = 4B$$

$$2 = B$$

$$5 = A+2$$

$$3 = A$$

$$\int \frac{3}{x-3}+\frac{2}{x+1}\,dx$$

$$3\ln|x-3|+2\ln|x+1|+C$$

This technique is called Partial Fractions

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Good News!

The AP Exam only requires non-repeating linear factors!

The more complicated methods of partial fractions are good to know, and you might see them in college, but they will not be on the AP exam or on my exam.

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Let's see another non repeating linear.

$$\int \frac{1}{x^2-5x+6} =$$

$$\frac{1}{x^2-5x+6} = \frac{1}{(x-3)(x-2)} = \frac{A}{(x-3)} + \frac{B}{(x-2)}$$

$$1 = A(x-2) + B(x-3)$$

How do we find A and B? **Find them.**

Hopefully you got A = 1 and B = -1

$$\int \frac{1}{x^2-5x+6} = \int \frac{1}{x-3} - \int \frac{1}{x-2}$$

2

$$\frac{6x+7}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2}$$

Repeated roots: we must use two terms for partial fractions.

$$6x+7 = A(x+2)+B$$

$$6x+7 = Ax+2A+B$$

$$6x = Ax \qquad 7 = 2A+B$$

$$6 = A \qquad 7 = 2\cdot 6 + B$$

$$7 = 12 + B$$

$$-5 = B$$

$$\frac{6}{x+2} - \frac{5}{(x+2)^2}$$

→

4

$$\frac{2x^3-4x^2-x-3}{x^2-2x-3}$$

If the degree of the numerator is higher than the degree of the denominator, use long division first.

$$\begin{array}{r} 2x \\ x^2-2x-3\overline{)\,2x^3-4x^2-x-3} \\ \underline{2x^3-4x^2-6x} \\ 5x-3 \end{array}$$

$$2x+\frac{5x-3}{x^2-2x-3}$$

(from example one)

$$2x+\frac{5x-3}{(x-3)(x+1)} = 2x+\frac{3}{(x-3)}+\frac{2}{(x+1)}$$

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