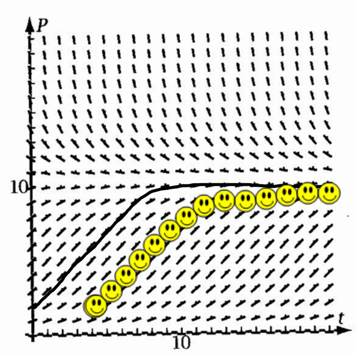
# 6.5 Logistic Growth

**What you'll learn about**

- How Populations Grow
- The Logistic Differential Equation
- Logistic Growth Models

**. . . and why**

Populations in the real world tend to grow logistically over extended periods of time.




Suppose that rabbits are introduced to a small uninhabited island in the Pacific.
Naturalists find the that differential equation for the population growth is

$$\frac{dP}{dt} = 0.038P(10.5 - P)$$

where P is in hundreds of rabbits and t is in months.
The slope field is shown above.

A) Suppose that 200 rabbits arrive at time t=0.
Graph the particular solution.
Use that solution to determine the number of rabbits in the population after 12 months.

B) Draw another particular solution if the 200 rabbits had been introduced at time t=4.
Use that solution to determine the number of rabbits in the population at time t=12 months.

C) Draw a third particular solution if 1800 rabbits had been introduced at time t=0.
Use your calculator to draw the slope field and approximate the number of rabbits in the population at time t=12 months for this solution.

D) With this initial condition, what is the major difference in population growth? Think of a real world reason to explain the horizontal asymptote each graph approaches.

## The Logistic Differential Equation

We have seen that the exponential growth at the beginning can be modeled by the differential equation

$$\frac{dP}{dt} = kP \text{ for some } k > 0.$$

If we want the growth rate to approach zero as $P$ approaches a maximal **carrying capacity** $M$, we can introduce a limiting factor of $M - P$:

$$\frac{dP}{dt} = kP(M - P)$$

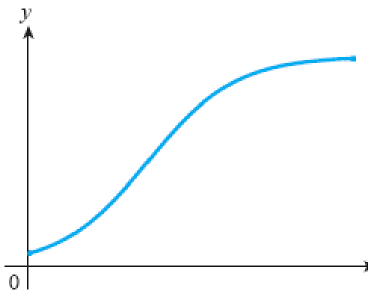



**Figure 6.13** A logistic curve.

### EXAMPLE 4

The growth rate of a population $P$ of bears in a newly established wildlife preserve is modeled by the differential equation $dP/dt = 0.008P(100 - P)$, where $t$ is measured in years.

**(a)** What is the carrying capacity for bears in this wildlife preserve?

**(b)** What is the bear population when the population is growing the fastest?

**(c)** What is the rate of change of the population when it is growing the fastest?

## The General Logistic Formula

The solution of the general logistic differential equation

$$\frac{dP}{dt} = kP(M - P)$$

is

$$P = \frac{M}{1 + Ae^{-(Mk)t}}$$

where $A$ is a constant determined by an appropriate initial condition. The **carrying capacity** $M$ and the **growth constant** $k$ are positive constants.

**EXAMPLE 5 Tracking a Moose Population**

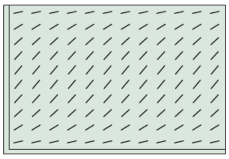
In 1985 and 1987, the Michigan Department of Natural Resources airlifted 61 moose from Algonquin Park, Ontario to Marquette County in the Upper Peninsula. It was originally hoped that the population $P$ would reach carrying capacity in about 25 years with a growth rate of

$$\frac{dP}{dt} = 0.0003P(1000 - P).$$

**(a)** According to the model, what is the carrying capacity?

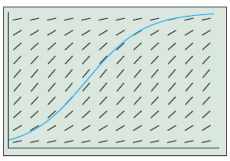
**(b)** With a calculator, generate a slope field for the differential equation.

**(c)** Solve the differential equation with the initial condition $P(0) = 61$ and show that it conforms to the slope field.



[0, 25] by [0, 1000]

**Figure 6.14** The slope field for the moose differential equation in Example 5.



[0, 25] by [0, 1000]

**Figure 6.15** The particular solution

$$P = \frac{1000}{1 + 15.393e^{-0.3t}}$$

conforms nicely to the slope field for $dP/dt = 0.0003P\,(1000 - P)$. (Example 5)

Assignment:
page 369/ Quick Review 5 to 10
23, 25, 31, 33, 34