Exponential Growth and Decay

6.4

What you'll learn about

- Separable Differential Equations
- Law of Exponential Change
- Continuously Compounded Interest
- Radioactivity
- Modeling Growth with Other Bases
- Newton's Law of Cooling

. . . and why

Understanding the differential equation $dy/dx = ky$ gives us new insight into exponential growth and decay.

Separable Differential Equations

DEFINITION Separable Differential Equation

A differential equation of the form $dy/dx = f(y)g(x)$ is called **separable.** We **separate the variables** by writing it in the form

$$\int \frac{1}{f(y)}\,dy = \int g(x)\,dx.$$

The solution is found by antidifferentiating each side with respect to its thusly isolated variable.

Procedure:
1) Separate the variables (including the dx, dy, etc.)
2) Integrate both sides
3) Find the constants (if a particular solution)
4) Solve for y

EXAMPLE 1 Solving by Separation of Variables

Solve for y if $dy/dx = (xy)^2$ and $y = 1$ when $x = 1$.

$$\frac{dy}{dx} = x^2y^2$$

$$\int y^{-2} \quad \int \frac{1}{y^2}\,dy = \int x^2\,dx$$

$$-y^{-1} = \frac{1}{3}x^3 + C$$

$$-\frac{1}{y} = \frac{1}{3}x^3 + C \longrightarrow \text{plug in } (1,1)$$

$$-1 = \frac{1}{3} + C$$

$$-\frac{4}{3} = C$$

$$-\frac{1}{y} = \frac{x^3+3C}{3}$$

$$y = -\frac{3}{x^3+3C}$$

$$-\frac{1}{y} = \frac{1}{3}x^3 - \frac{4}{3}$$

$$-\frac{1}{y} = \frac{x^3-4}{3}$$

$$-y = \frac{3}{x^3-4}$$

$$y = \frac{-3}{x^3-4}$$

Example 1: Find the solution for the following differential equation when y(1) = 2.

$$\frac{dy}{dx} = \frac{x}{y}$$

$(1, 2)$

$y\,dy = x\,dx$

$\frac{1}{2}y^2 = \frac{1}{2}x^2 + C \longrightarrow 2 = \frac{1}{2} + c$

$c = \frac{3}{2}$

$y^2 = x^2 + 2C$

$\frac{1}{2}y^2 = \frac{1}{2}x^2 + \frac{3}{2}$

$y = \pm\sqrt{x^2 + 2C}$

$y = \pm\sqrt{x^2 + 3}$

$y = \pm\sqrt{x^2 + A}$

A Culture of Bacteria...
Exponential Growth

One thousand bacteria are started in a certain culture and the number of bacteria B, increase at a rate porportional to the number present. Set up and solve the differential equation which represents this situation when B = 3000 when time t = 8.

$(0, 1000)$ $(8, 3000)$

$$\frac{dB}{dt} = BK$$

$$\int \frac{dB}{B} = \int k\, dt$$

$$\ln|B| = kt + C$$

$C = \ln 1000$

$$|B| = e^{kt+C}$$

$$B = \pm e^{kt} \cdot e^{C}$$

$$B = \pm e^{C} e^{kt}$$

$$B = Ae^{kt}$$

$$1000 = Ae^{k(0)}$$

$$3000 = Ae^{(8)k}$$

$$A = 1000$$

$$3000 = 1000e^{8k}$$

$$3 = e^{8k}$$

$$\ln 3 = 8k$$

$$\frac{\ln 3}{8} = k$$

Assign:
page 357/ ~~Quick Review 1 to 10~~
1 to 13 odd