Name______________________________

10.1: Infinite Series

In short, a finite sum of real numbers always produces a real number (the result of a finite number of binary additions), but an infinite sum of real numbers is something else entirely.

What is a Series vs a Sequence?

A Series is the sum of a Sequence

What is the definition of an infinite series?

expression of form $a_1 + a_2 + a_3 + \cdots a_n + \cdots$ $\sum_{k=1}^{\infty} a_k$

What is the difference between a series converging and a series diverging?

If the sequence of partial sums has a limit then Series converges, otherwise Diverges.

Example 1: Does the series 1-1+1-1+1-1+… converge?

sequ. of partial sums: 1, 0, 1, 0, 1, 0 since this div. then Series div.

Example 2: Why does the series $3/10^n$ for n > 1 converge to 1/3

sequ. of partial sums: .3, .33, .333, .3333, ... → $.\overline{3}$ or $\frac{1}{3}$

Thus partial sums converge to $\frac{1}{3}$.

What is the definition and formula of a geometric series?

$$\sum_{n=1}^{\infty} a_1 r^{n-1} = a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \cdots$$

The interval -1 < r < 1 is the interval of convergence.

Example 3: Tell whether each series converges or diverges. If it converges, give its sum. (Look in book for series a-d)

a) $\sum_{n=1}^{\infty} 3\left(\frac{1}{2}\right)^{n-1}$

$-1 < \left(\frac{1}{2}\right) < 1$

$S_\infty = \frac{3}{1-\frac{1}{2}} = \frac{3}{\frac{1}{2}} = 6$

b) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots -\frac{1}{2}^{n-1}$

$r = -\frac{1}{2}$

$S_\infty = \frac{1}{1 - -\frac{1}{2}} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$

c) $\sum_{k=0}^{\infty} \left(\frac{3}{5}\right)^k$

$r = \frac{3}{5}$ $-1 < \frac{3}{5} < 1$ $S_\infty = \frac{1}{1-\frac{3}{5}}$

$5/2$

d) $\frac{\pi}{2} + \frac{\pi^2}{4} + \frac{\pi^3}{8}$

$r = \frac{\pi}{2} > 1$

∴ D.V.

If |x| < 1, then the geometric series formula assures us that $\boxed{1 + x + x^2 + \cdots x^n + \cdots} = \frac{1}{1-x}$ $r = x$

$\sum_{n=0}^{\infty} x^n$ ratio: x $\frac{a_1}{1-r} = \frac{1}{1-x}$

What is the definition of a power series?

expression of form $\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + \cdots + c_n x^n + \cdots$

is a power series centered at $x=0$

$\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \cdots$ is power series centered @ $x=a$

Example 4: Given that 1/(1-x) is represented by the power series $1 + x + x^2 + \ldots + x^n + \ldots$ on the interval (-1,1), find a power series to represent $1/(1-x)^2$.

$$\frac{d}{dx}\frac{1}{1-x} = \frac{d}{dx}\, 1 + x + x^2 + x^3 + \ldots$$

$$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + \ldots$$

What is theorem #1 of term-by-term differentiation?

If $f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \ldots$ and converges for $|x-a| < R$, then the series

$$f'(x) = \sum_{n=0}^{\infty} n c_n (x-a)^{n-1} = c_1 + 2c_2(x-a) + 3c_3(x-a)^2 + \ldots$$

Theorem #2 of term-by-term integration?

If $f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \ldots$ and converges for $|x-a| < R$, then the series

$$\sum_{n=0}^{\infty} c_n \frac{(x-a)^{n+1}}{n+1} = c_0(x-a) + c_1\frac{(x-a)^2}{2} + c_2\frac{(x-a)^3}{3} + \ldots$$

Example 5: Given that $1/1\text{-}x = 1 + x + x^2 + x^3 + \ldots + (-x)^n + \ldots$, $-1 < x < 1$, find a power series to represent ln(1+x).

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots$$

$$\frac{1}{1-(-x)} = \frac{1}{1+x} = 1 + (-x) + (-x)^2 + (-x)^3 + \ldots$$

$$= 1 - x + x^2 - x^3 + \ldots$$

$$\int \frac{1}{1+x} = \int 1 - x + x^2 - x^3 + \ldots$$

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \ldots$$