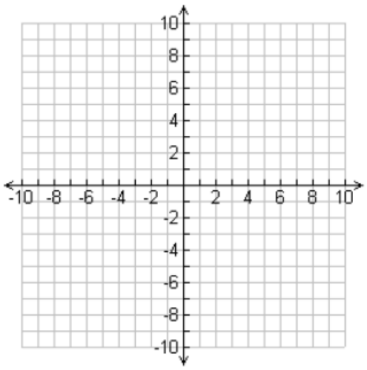


31. **BIKE COSTS** You buy a new mountain bike for $200. The value of the bike decreases by 25% each year.

a. Write a model giving the mountain bike's value y (in dollars) after t years. Use the model to estimate the value of the bike after 3 years.

b. Graph the model.

c. Estimate when the value of the bike will be $100.

$y = 200(1 - .25)^t$
$= 200(.75)^t$

$y_1 = 200(.75)^t$
$y_2 = 100$

put in calculator and find intersection.

3. A scientist observes 27 bacteria under a microscope. It is expected to grow at a rate of 22% an hour. How many bacteria would be expected after 8 hours?

$y = 27(1 + .22)^8$
$= 27(1.22)^8$
$\doteq 133$ bacteria

5. If a patient takes a 50 mg dose of medication and it is leaves the bloodstream at a rate of 11% per hour, how much remains in the blood after 4.5 hours?

$y = 50(1 - .11)^{4.5}$
$= 50(.89)^{4.5}$
$\doteq 30$ mg

4.3 Use Functions Involving *e*

Before You studied exponential growth and decay functions.
Now You will study functions involving the natural base *e*.
Why? So you can model visibility underwater, as in Ex. 59.

F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Key Vocabulary
- **natural base *e***

The history of mathematics is marked by the discovery of special numbers such as π and i. Another special number is denoted by the letter e. The number is called the **natural base *e*** or the *Euler number* after its discoverer, Leonhard Euler (1707–1783). The expression $\left(1 + \frac{1}{n}\right)^n$ approaches e as n increases.

Use your roster number * 100 for n in the above expression. Let's hear what you got.

KEY CONCEPT For Your Notebook

The Natural Base e

The natural base e is irrational. It is defined as follows:

As n approaches $+\infty$, $\left(1+\frac{1}{n}\right)^n$ approaches $e \approx 2.718281828$.

GUIDED PRACTICE for Examples 1 and 2

Simplify the expression.

1. $e^7 \cdot e^4$

$e^{7+4} = e^{11}$

Exponent rules apply to common bases, including e.

2. $2e^{-3} \cdot 6e^5$

$= 2 \cdot 6 \cdot e^{-3} \cdot e^5$

$= 12e^2$

GUIDED PRACTICE for Examples 1 and 2

Simplify the expression.

3. $\frac{24e^8}{4e^5} = 6e^{8-5} = 6e^3$

4. $(10e^{-4x})^3$

$= (10)^3 (e^{-4x})^3 = 1000e^{-12x} = \frac{1000}{e^{12x}}$

5. Use a calculator to evaluate $e^{3/4}$.

$e \wedge (3/4) \approx 2.12$

e is on the calc face twice. Use either one. Left side is easier though.

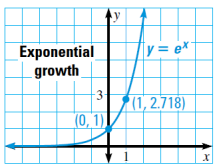
KEY CONCEPT For Your Notebook

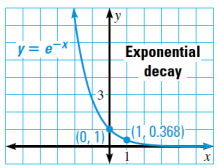
Natural Base Functions

A function of the form $y = ae^{rx}$ is called a *natural base exponential function*.

- If $a > 0$ and $r > 0$, the function is an exponential growth function.
- If $a > 0$ and $r < 0$, the function is an exponential decay function.

The graphs of the basic functions $y = e^x$ and $y = e^{-x}$ are shown below.





GUIDED PRACTICE for Examples 3 and 4

Graph the function. State the domain and range.

7. $f(x) = \frac{1}{2}e^{-x} + 1$

What is the growth factor?

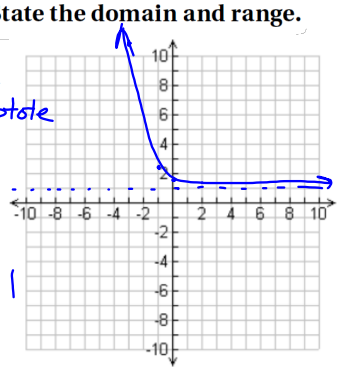
$(e^{-1}) \approx .368$

asymptote

x	y
-3	11
-1	2.3
0	1.5
2	1.06

D: R

R: $R > 1$



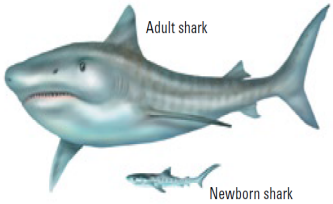
EXAMPLE 4 **Solve a multi-step problem**

BIOLOGY The length ℓ (in centimeters) of a tiger shark can be modeled by the function

$$\ell = 337 - 276e^{-0.178t}$$

where t is the shark's age (in years).

- Graph the model.
- Use the graph to estimate the length of a tiger shark that is 3 years old.



$\ell = 337 - 276e^{-.178(3)}$

$\ell \approx 175 \text{ cm}$