$y'' = 0$

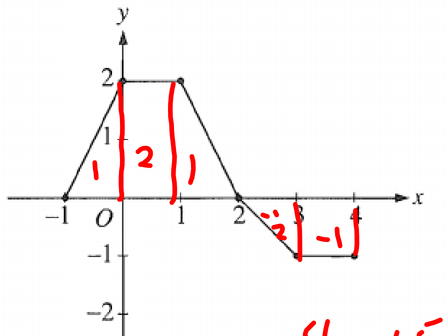
1. What is the x-coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^3 + 5x^2 + 24$?

(A) 5 (B) 0 (C) $-\frac{10}{3}$ (D) -5 (E) -10

$y' = x^2 + 10x$

$y'' = 2x + 5$

$x = -5$



$4 - 1.5 =$

2. The graph of a piecewise-linear function f, for $-1 \le x \le 4$, is shown above. What is the value of $\int_{-1}^{4} f(x)\,dx$?

(A) 1 (B) 2.5 (C) 4 (D) 5.5 (E) 8

3. $\int_1^2 \frac{1}{x^2}\,dx =$

(A) $-\frac{1}{2}$ (B) $\frac{7}{24}$ (C) $\frac{1}{2}$ (D) 1 (E) $2 \ln 2$

$\int_1^2 x^{-2}\,dx$ $\quad -1x^{-1}\Big|_1^2$

$-\frac{1}{x}\Big|_1^2 \quad -\frac{1}{2}+\frac{1}{1}=$

4. If f is continuous for $a \le x \le b$ and differentiable for $a < x < b$, which of the following could be false?

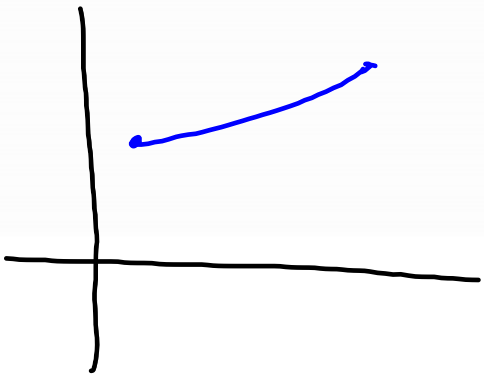
(A) $f'(c) = \frac{f(b) - f(a)}{b - a}$ for some c such that $a < c < b$. MVT

(B) $f'(c) = 0$ for some c such that $a < c < b$.

(C) f has a minimum value on $a \le x \le b$.

(D) f has a maximum value on $a \le x \le b$.

(E) $\int_a^b f(x)\,dx$ exists.



5. $\int_0^x \sin t\, dt =$

(A) $\sin x$ (B) $-\cos x$ (C) $\cos x$ (D) $\cos x - 1$ (E) $1 - \cos x$

$-\cos t \Big|_0^x$

$-\cos x - -\cos 0$

$-\cos x + 1$

$2x + x\frac{dy}{dx} + 1y = 0$

6. If $x^2 + xy = 10$, then when $x = 2$, $\frac{dy}{dx} =$

$2^2 + 2(y) = 10 \quad y = 3$

(A) $-\frac{7}{2}$ (B) -2 (C) $\frac{2}{7}$ (D) $\frac{3}{2}$ (E) $\frac{7}{2}$

$2x + x\frac{dy}{dx} + 1y = 0$

$\boxed{\frac{dy}{dx} = \frac{-2x - y}{x}}$

$\frac{-2(2) - 3}{2}$

7. $\int_1^e \left(\frac{x^2-1}{x}\right) dx =$

(A) $e - \frac{1}{e}$ (B) $e^2 - e$ (C) $\frac{e^2}{2} - e + \frac{1}{2}$ (D) $e^2 - 2$ (E) $\frac{e^2}{2} - \frac{3}{2}$

$$\int_1^e \left(x - \frac{1}{x}\right) dx$$

$$\frac{1}{2}x^2 - \ln x \Big|_1^e$$

$$\frac{1}{2}e^2 - \ln e - \left[\frac{1}{2} - \ln 1^{0}\right]$$

$\ln e = 1$ $e^1 = e$

$\ln 1 = 0$ $e^0 = 1$

$$\frac{e^2}{2} - 1 - \frac{1}{2}$$

8. Let f and g be differentiable functions with the following properties:

(i) $g(x) > 0$ for all x

(ii) $f(0) = 1$

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

(A) $f'(x)$ (B) $g(x)$ (C) e^x (D) 0 (E) 1

$$h'(x) = f'(x)g(x) + g'(x)f(x) = f(x)g'(x)$$

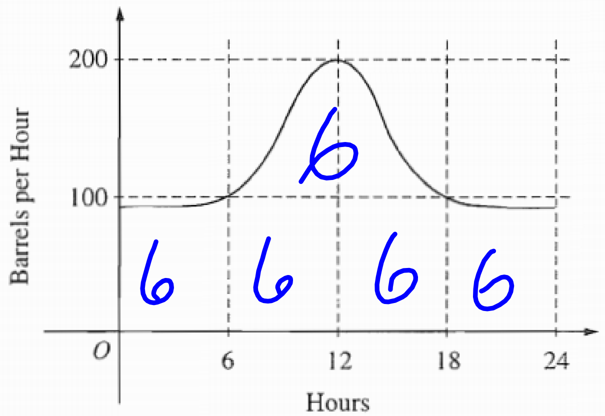
$$\downarrow$$

$$f'(x)g(\cancel{x}) = 0$$

$$f'(x) = 0$$

$f(x) = \text{constant}$

$f(x) = 1$



9. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

$f'(x)$

10. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \frac{x^2 - 2}{x - 1}$?

(A) -2 (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) 2 (E) 6

$$f'(x) = \frac{(x-1)2x - (x^2-2)(1)}{(x-1)^2}$$

$f'(2)$

$$\frac{4-2}{1} = 2$$

11. If f is a linear function and $0 < a < b$, then $\int_a^b f''(x)\,dx =$

(A) 0 (B) 1 (C) $\frac{ab}{2}$ (D) $b - a$ (E) $\frac{b^2 - a^2}{2}$

$f''(x) = 0$

$$\int_a^b f''(x)\,dx$$

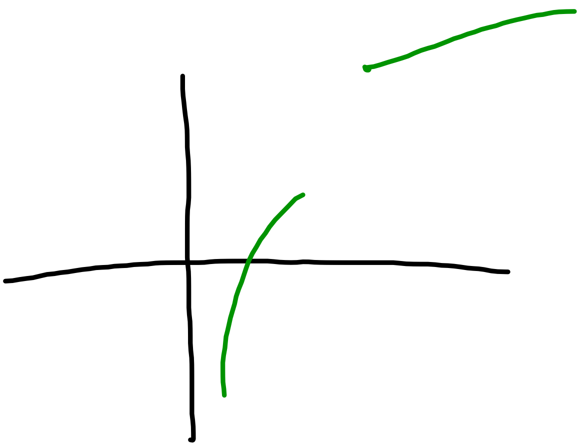
$$\int_a^b 0\,dx = 0$$

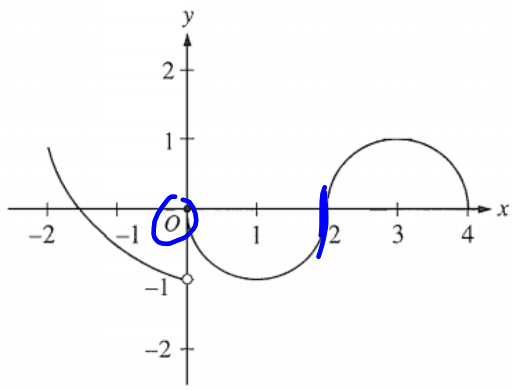
12. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim_{x \to 2} f(x)$ is

$\ln 2$

$4 \ln 2$

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent





13. The graph of the function f shown in the figure above has a vertical tangent at the point (2, 0) and horizontal tangents at the points (1, −1) and (3, 1). For what values of x, $-2 < x < 4$, is f not differentiable?

(A) 0 only (B) 0 and 2 only (C) 1 and 3 only (D) 0, 1, and 3 only (E) 0, 1, 2, and 3

14. A particle moves along the x-axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

$x'(t) = 2t - 6$

$t = 3$

FTC

15. If $F(x) = \int_0^x \sqrt{t^3 + 1}\, dt$, then $F'(2) =$

(A) -3 (B) -2 (C) 2 (D) 3 (E) 18

$$F(x) = \int_0^x \sqrt{t^3+1}\, dt$$

$$F'(x) = \sqrt{x^3+1}$$

$$F'(2) = \sqrt{2^3+1}$$

$$\sqrt{9} = 3$$

16. If $f(x) = \sin(e^{-x})$, then $f'(x) =$

(A) $-\cos(e^{-x})$

(B) $\cos(e^{-x}) + e^{-x}$

(C) $\cos(e^{-x}) - e^{-x}$

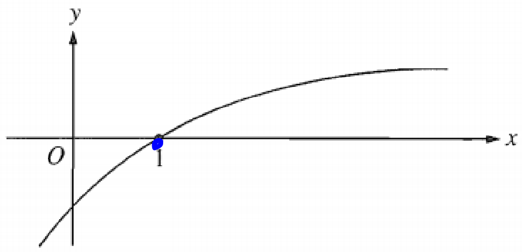
(D) $e^{-x}\cos(e^{-x})$

(E) $-e^{-x}\cos(e^{-x})$

$$d\, e^u = e^u\, du$$

$$f'(\sin u) = \cos u\, du$$

$$\cos(e^{-x})(e^{-x})(-1)$$



17. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$
(B) $f(1) < f''(1) < f'(1)$
(C) $f'(1) < f(1) < f''(1)$
(D) $f''(1) < f(1) < f'(1)$
(E) $f''(1) < f'(1) < f(1)$

$f(1)=0$

$f'(1)>0$ (increasing

$f''(1)<0$ concave down

18. An equation of the line tangent to the graph of $y = x + \cos x$ at the point $\boxed{(0, 1)}$ is

(A) $y = 2x + 1$ (B) $y = x + 1$ (C) $y = x$ (D) $y = x - 1$ (E) $y = 0$

$y = x + \cos x$

$y' = 1 - \sin x$

$y'(0) = 1 - \sin 0$

$y'(0) = 1$

$m = 1$ $(0,1)$

$y - 1 = 1(x - 0)$

$\boxed{y = x + 1}$

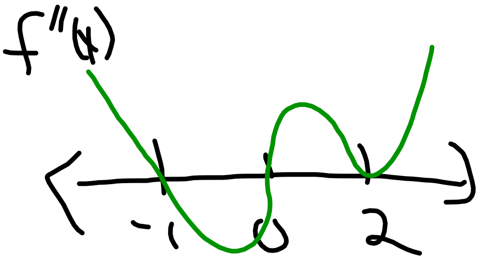
$0 \quad -1$

19. If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when $x =$

(A) -1 only (B) 2 only (C) -1 and 0 only (D) -1 and 2 only (E) -1, 0, and 2 only

$f''(x) = 0$

$f''(x) = x(x+1)\boxed{(x-2)^2}$



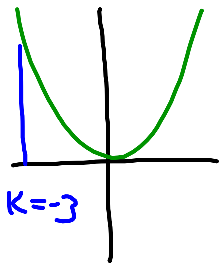
$x = 0$

$x = -1$

$\cancel{x = 2}$

20. What are all values of k for which $\int_{-3}^{k} x^2\,dx = 0$?

(A) -3 (B) 0 (C) 3 (D) -3 and 3 (E) -3, 0, and 3



$\frac{1}{3}x^3 \Big|_{-3}^{K}$

$\frac{1}{3}\left[K^3 - (-3)^3\right] = 0$

$\frac{1}{3}\left[K^3 + 27\right] = 0$

$K^3 = -27 \quad K = -3$

21. If $\frac{dy}{dt} = ky$ and k is a nonzero constant, then y could be

(A) $2e^{kty}$ (B) $2e^{kt}$ (C) $e^{kt} + 3$ (D) $kty + 5$ (E) $\frac{1}{2}ky^2 + \frac{1}{2}$

exponential

$\frac{dy}{dt} = ky$

$y = y_0 e^{kt}$

22. The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

(A) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$

(B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(C) $(0, \infty)$

(D) $(-\infty, 0)$

(E) $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

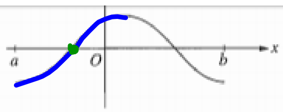
$f'(x) > 0$

$f'(x) = 4x^3 + 2x$

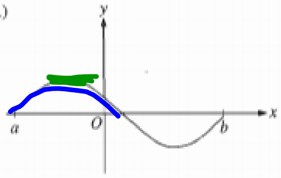
$4x^3 + 2x > 0$

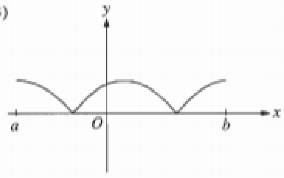
$2x(2x^2+1) > 0$

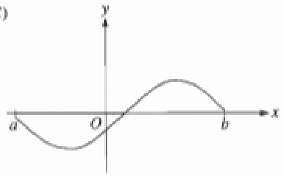
$x > 0$

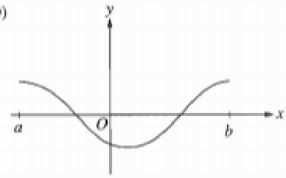


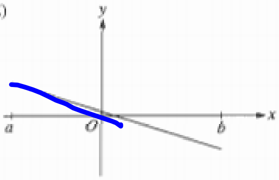
23. The graph of f is shown in the figure above. Which of the following could be the graph of the derivative of f?

(A)


(B)


(C)


(D)


(E)


Part A

$t=3$

24. The maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

(A) 9 (B) 12 (C) 14 (D) 21 (E) 40

endpoint

$a(t)$ is the function I want to maximize

$\to a'(t)=0$

endpoints

$a(t) = 3t^2 - 6t + 12$

$a'(t) = 6t - 6$

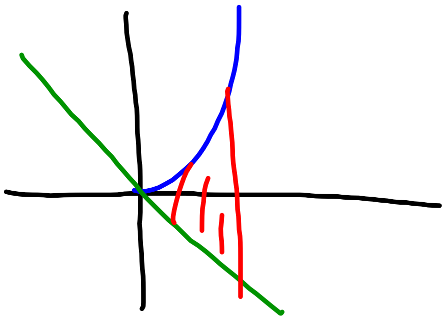
$t = 1$

$a(0) = 12$

$a(3) = 3(3^2) - 6(3) + 12$

25. What is the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 2$?

(A) $\frac{2}{3}$ (B) $\frac{8}{3}$ (C) 4 (D) $\frac{14}{3}$ (E) $\frac{16}{3}$



$$\int_0^2 (x^2+x)\,dx$$

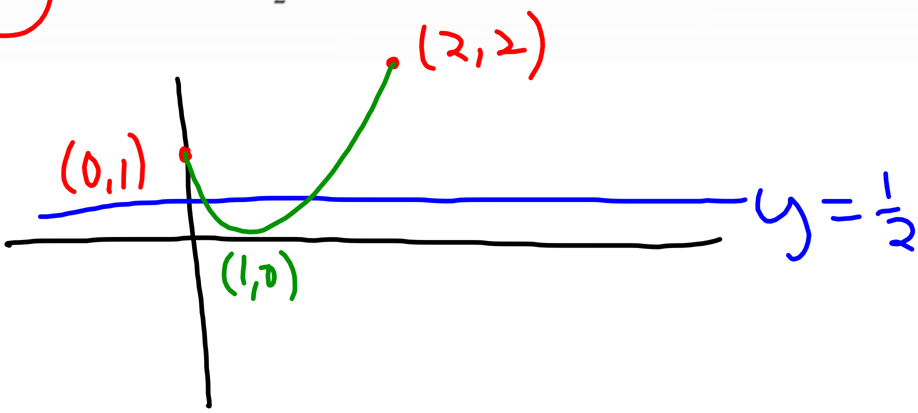
$$\frac{1}{3}x^3+\frac{1}{2}x^2\Big|_0^2$$

$$\frac{8}{3}+\frac{6}{3}=\frac{14}{3}$$

x	0	1	2
$f(x)$	1	k	2

26. The function f is continuous on the closed interval [0, 2] and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval [0, 2] if $k =$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3



27. What is the average value of $y = x^2\sqrt{x^3+1}$ on the interval $[0, 2]$?

(A) $\frac{26}{9}$ (B) $\frac{52}{9}$ (C) $\frac{26}{3}$ (D) $\frac{52}{3}$ (E) 24

$$\boxed{\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}} \quad \frac{1}{3}\,\frac{1}{2}\int_0^2 3x^2\sqrt{x^3+1}\,dx \qquad \boxed{u = x^3+1}$$

$$du = 3x^2\,dx$$

$$\frac{1}{6}\int_1^9 u^{\frac{1}{2}}\,du$$

$$u(0)=1$$

$$u(2)=9$$

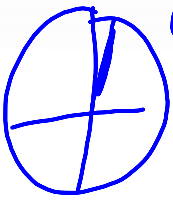
$$\frac{1}{6}\,\frac{2}{3}\,u^{\frac{3}{2}}\Big|_1^9$$

$$\frac{1}{9}u^{\frac{3}{2}}\Big|_1^9 \qquad \frac{1}{9}\left[9^{\frac{3}{2}} - 1^{\frac{3}{2}}\right]$$

$$\frac{1}{9}[27-1] = \frac{26}{9}$$

28. If $f(x) = \tan(2x)$, then $f'\left(\frac{\pi}{6}\right) =$

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 4 (D) $4\sqrt{3}$ (E) 8



$\cos = \frac{1}{2}$

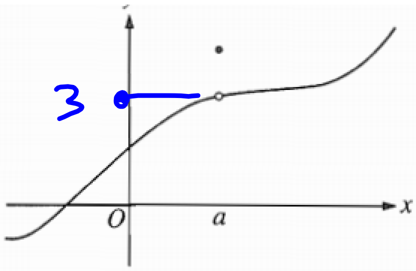
$\sec = 2$

$$f(x) = \tan(2x)$$

$$f'(x) = 2\sec^2 2x \qquad 2\sec^2\left(\frac{\pi}{3}\right)$$

$$2(4) = 8$$

$2(2) - $



76. The graph of a function f is shown above. Which of the following statements about f is false?

(A) f is continuous at $x = a$.

(B) f has a relative maximum at $x = a$.

(C) $x = a$ is in the domain of f.

(D) $\lim_{x \to a^+} f(x)$ is equal to $\lim_{x \to a^-} f(x)$.

(E) $\lim_{x \to a} f(x)$ exists.

77. Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

(A) -0.701
(B) -0.567
(C) -0.391
(D) -0.302
(E) -0.258

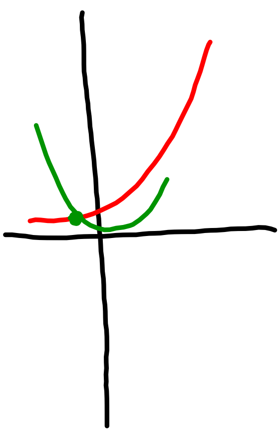
Slopes are equal

$f'(x) = 6e^{2x}$ $\quad$ $g'(x) = 18x^2$

$y_1 = 6e^{2x}$

$y_2 = 18x^2$

intersection



78. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

(A) $-(0.2)\pi C$

(B) $-(0.1)C$

(C) $-\frac{(0.1)C}{2\pi}$

(D) $(0.1)^2 C$

(E) $(0.1)^2 \pi C$

$A = \pi r^2$

$A = \pi\left(\frac{C}{2\pi}\right)^2$

$A = \frac{1}{4\pi} C^2$

$\frac{dr}{dt} = -0.1$

$\frac{dA}{dt} = ?$

$C = 2\pi r$

$r = \frac{C}{2\pi}$

$C = 2\pi r$

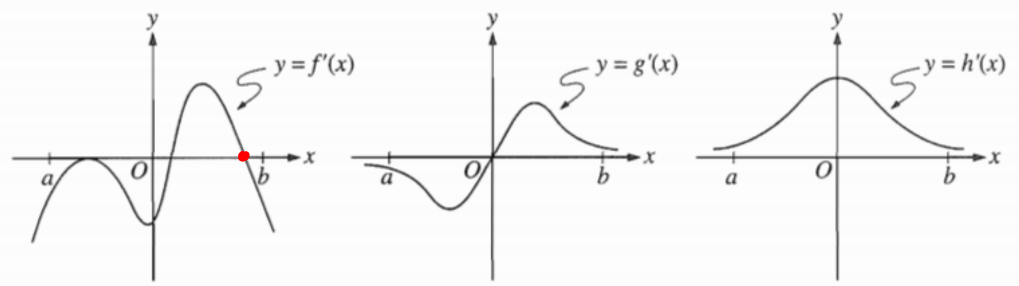
$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$

$\frac{dA}{dt} = \frac{1}{2\pi} C \frac{dC}{dt}$

$\frac{dA}{dt} = \frac{1}{2\pi} \cdot C\, 2\pi(-.1)$

$\frac{dA}{dt} = -.1C$

Part B



79. The graphs of the derivatives of the functions f, g, and h are shown above. Which of the functions f, g, or h have a relative maximum on the open interval $a < x < b$?

(A) f only

(B) g only

(C) h only

(D) f and g only

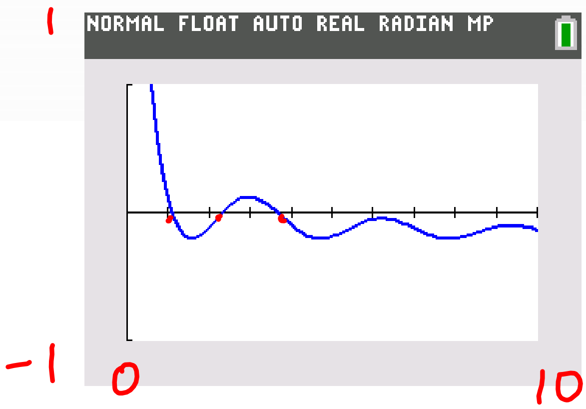
(E) f, g, and h

increasing to decreasing

80. The first derivative of the function f is given by $f'(x) = \dfrac{\cos^2 x}{x} - \dfrac{1}{5}$. How many critical values does f have on the open interval (0, 10)?

$f' = 0$

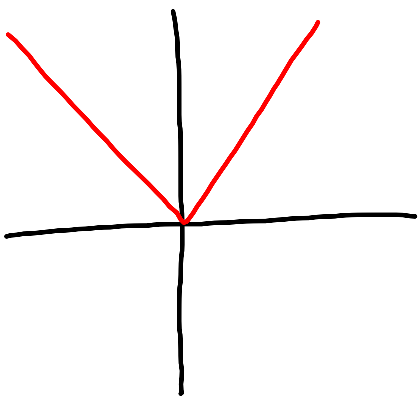
(A) One
(B) Three
(C) Four
(D) Five
(E) Seven



81. Let f be the function given by $f(x) = |x|$. Which of the following statements about f are true?

I. f is continuous at $x = 0$.
II. f is differentiable at $x = 0$.
III. f has an absolute minimum at $x = 0$.

(A) I only (B) II only (C) III only (D) I and III only (E) II and III only



82. If f is a continuous function and if $F'(x) = f(x)$ for all real numbers x, then $\int_1^3 f(2x)dx =$

(A) $2F(3) - 2F(1)$

(B) $\frac{1}{2}F(3) - \frac{1}{2}F(1)$

(C) $2F(6) - 2F(2)$

(D) $F(6) - F(2)$

(E) $\frac{1}{2}F(6) - \frac{1}{2}F(2)$

$\int_1^3 f(2x)dx$

$\int_a^b \cos 2x\, dx$

$\frac{1}{2}\sin 2b - \frac{1}{2}\sin 2a$

83. If $a \neq 0$, then $\lim_{x\to a} \frac{x^2 - a^2}{x^4 - a^4}$ is

(A) $\frac{1}{a^2}$ (B) $\frac{1}{2a^2}$ (C) $\frac{1}{6a^2}$ (D) 0 (E) nonexistent

$\lim_{x\to a} \frac{\cancel{(x^2-a^2)}}{\cancel{(x^2-a^2)}(x^2+a^2)}$ $\lim_{x\to a} \frac{1}{x^2+a^2}$

$\frac{1}{a^2+a^2} = \frac{1}{2a^2}$

84. Population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

(A) 0.069 (B) 0.200 (C) 0.301 (D) 3.322 (E) 5.000

$$\frac{dy}{dt} = ky$$

$$y = y_0 e^{kt}$$

$$\frac{y}{y_0} = 2$$

$$2 = e^{kt}$$

$$\frac{\ln 2}{10} = \frac{k(10)}{10}$$

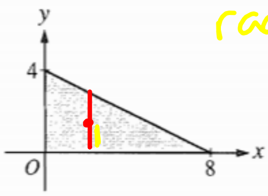
x	2	5	7	8
$f(x)$	10	30	40	20

20

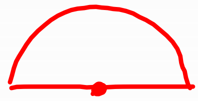
85. The function f is continuous on the closed interval [2, 8] and has values that are given in the table above. Using the subintervals [2, 5], [5, 7], and [7, 8], what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$?

(A) 110 (B) 130 (C) 160 (D) 190 (E) 210

$$3(20) + 2(35) + 1(30)$$



radius $= \frac{1}{2}$ y-value



86. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x + 2y = 8$, as shown in the figure above. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

(A) 12.566 (B) 14.661 (C) 16.755 (D) 67.021 (E) 134.041

$A = \frac{1}{2}\pi r^2$

$2y = -x + 8$

$r = -\frac{1}{4}x + 2$

$y = -\frac{1}{2}x + 4$

$\frac{1}{2}y = -\frac{1}{4}x + 2$

$\frac{1}{2}\pi \int_0^8 \left(-\frac{1}{4}x + 2\right)^2 dx$

point slope

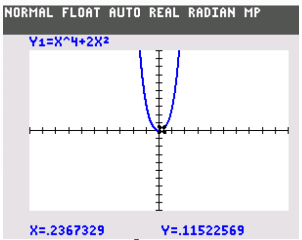
87. Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?

(A) $y = 8x - 5$
(B) $y = x + 7$
(C) $y = x + 0.763$
(D) $y = x - 0.122$
(E) $y = x - 2.146$

Slope $= 1$

$f'(x) = 1$

$f'(x) = 4$



$y_1 = 4x^3 + 4x$

$y_2 = 1$

intersectin

$x = .2367329$

$y = .11522$

$y - .11522 = 1(x - .2367329)$

$+.11522 \qquad .11522$

$\boxed{y = x - .122}$

88. Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1) = 0$, then $F(9) =$

(A) 0.048 (B) 0.144 (C) 5.827 (D) 23.308 (E) 1,640.250

$$\int_1^9 \frac{(\ln x)^3}{x}\,dx$$

NORMAL FLOAT AUTO REAL RADIAN MP

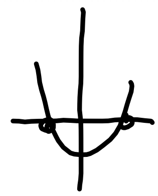
$\int_1^9 \left(\frac{(\ln(X))^3}{X}\right)dX$

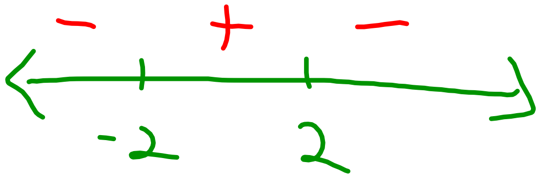
5.826903176

89. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = (x^2 - 4)g(x)$, which of the following is true?

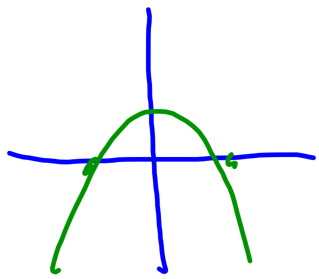
(A) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
(B) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
(C) f has relative minima at $x = -2$ and at $x = 2$.
(D) f has relative maxima at $x = -2$ and at $x = 2$.
(E) It cannot be determined if f has any relative extrema.

$y = (x^2 - 4)$



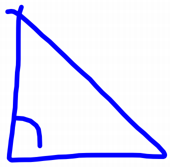


$f'(x) =$



90. If the base b of a triangle is increasing at a rate of 3 inches per minute while its height h is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

(A) A is always increasing.
(B) A is always decreasing.
(C) A is decreasing only when $b < h$.
(D) A is decreasing only when $b > h$.
(E) A remains constant.



$$A = \frac{1}{2} b \cdot h$$

$$\frac{db}{dt} = 3$$

$$\frac{dA}{dt} = \frac{1}{2}\left[b\frac{dh}{dt} + h\frac{db}{dt}\right]$$

$$\frac{dh}{dt} = -3$$

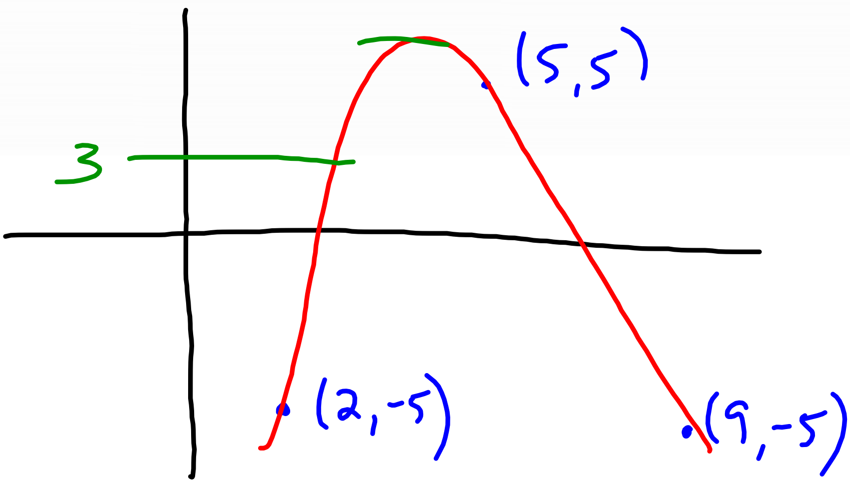
$$\frac{dA}{dt} = \frac{1}{2}\left[b(-3) + h(3)\right]$$

Smooth continuous curve

91. Let f be a function that is differentiable on the open interval (1, 10). If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, which of the following must be true?

I. f has at least 2 zeros. IVT
II. The graph of f has at least one horizontal tangent.
III. For some c, $2 < c < 5$, $f(c) = 3$. IVT

(A) None
(B) I only
(C) I and II only
(D) I and III only
(E) I, II and III



92. If $0 \le k < \frac{\pi}{2}$ and the area under the curve $y = \cos x$ from $x = k$ to $x = \frac{\pi}{2}$ is 0.1, then $k =$

(A) 1.471 (B) 1.414 (C) 1.277 (D) 1.120 (E) 0.436

$$\int_{k}^{\frac{\pi}{2}} \cos x = 0.1$$

$$\sin x \Big|_{k}^{\frac{\pi}{2}}$$

$$\sin \tfrac{\pi}{2} - \sin k = 0.1$$

$$1 - \sin k = 0.1$$

$$-\sin k = -.9$$

$$k = \sin^{-1}(.9)$$